Math 10A with Professor Stankova
Worksheet, Discussion #9; Friday, 9/15/2017
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Example

1. Find $\lim_{x \to 0^-} x^{-1} e^{x^{-2}}$.

Solution: As $x \to 0^+$, we know that $x^{-1} \to -\infty$ and $x^{-2} \to \infty$ so $x^{-1}e^{x^{-2}} \to -\infty \cdot e^{\infty} = -\infty$.

Problems

2. Find $\lim_{x \to 2\pi^-} x \csc(x)$.

Solution: As $x \to 2\pi^-$, we know that $\sin(x) \to 0^-$ and hence $x \csc(x) = \frac{x}{\sin(x)} \to -\infty$.

3. Find $\lim_{x \to 2\pi^+} x \csc(x)$.

Solution: As $x \to 2\pi^+$, we know that $\sin(x) \to 0^+$ and hence $x \csc(x) = \frac{x}{\sin(x)} \to \infty$.

4. Find $\lim_{x \to 2^-} \dfrac{x^2 - 2x - 8}{-(x^2 - 5x + 6)}$.

Solution: Factoring, we have that $\dfrac{x^2 - 2x - 8}{-(x^2 - 5x + 6)} = \dfrac{(x-4)(x+2)}{-(x-2)(x-3)}$. As $x \to 2^-$, we get $\frac{-8}{0}$ and so we need to care about the sign. Since $x < 2$, the signs are $\frac{- \cdot +}{- \cdot - \cdot -} = +$ so the limit is $+\infty$.

5. Find $\lim_{x \to 2^+} \dfrac{x^2 - 2x - 8}{-(x^2 - 5x + 6)}$.

Solution: Factoring, we have that $\dfrac{x^2 - 2x - 8}{-(x^2 - 5x + 6)} = \dfrac{(x-4)(x+2)}{-(x-2)(x-3)}$. As $x \to 2^+$, we get $\frac{-8}{0}$ and so we need to care about the sign. Since $x > 2$, the signs are $\frac{- \cdot +}{- \cdot + \cdot -} = -$ so the limit is $-\infty$.

6. Find $\lim_{x \to 3^-} \frac{\sqrt{x}}{(x-3)^4}$.

> **Solution:** As $x \to 3^-$, we have that $(x-3)^4 \to 0^+$ and so the limit is ∞.

7. Find $\lim_{x \to 3^+} \frac{\sqrt{x}}{(x-3)^4}$.

> **Solution:** As $x \to 3^+$, we have that $(x-3)^4 \to 0^+$ and so the limit is ∞.

Example

1. Find y' if $x^3 + y^3 = 4$.

> **Solution:** Taking the derivative of both sides, we have that $3x^2 + 3y^2 \cdot y' = 0$ and hence $y' = \frac{-3x^2}{3y^2} = \frac{-x^2}{y^2}$.

2. Find y' if $e^{xy} = e^{4x} - e^{5y}$.

> **Solution:** We take the derivative to get that $4e^{4x} - 5e^{5y}y' = e^{xy}(xy)' = e^{xy}(xy' + y)$. Bringing all the y' to one side, we have $4e^{4x} - ye^{xy} = 5e^{5y}y' + xe^{xy}y'$ so $y' = \dfrac{4e^{4x} - ye^{xy}}{5e^{5y} + xe^{xy}}$.

Problems

3. Find y' if $(x-y)^2 = x + y - 1$.

> **Solution:** Taking the derivative gives $2(x-y)y' = 1 + y'$ so $y' = \dfrac{1}{2x - 2y - 1}$.

4. Find y' if $y = \sin(3x + 4y)$.

> **Solution:** Taking the derivative gives $y' = \cos(3x+4y)(3x+4y)' = \cos(3x+4y)(3+4y')$. Thus, $y' = \dfrac{3\cos(3x+4y)}{1 - 4\cos(3x+4y)}$.

5. Find y' if $y = x^2y^3 + x^3y^2$.

Solution: Taking the derivative gives $y' = 2xy^3 + 3x^2y^2y' + 3x^2y^2 + 2x^3yy'$ and hence $y' = \dfrac{2xy^2 + 3x^2y^2}{1 - 3x^2y^2 - 2x^3y}$.

6. Find y' if $\cos^2 x + \cos^2 y = \cos(2x + 2y)$.

Solution: Taking the derivative gives $-2\cos x \sin x - 2y' \cos y \sin y = -2\sin(2x + 2y) - 2y' \sin(2x + 2y)$ and so

$$y' = \frac{\cos x \sin x - \sin(2x + 2y)}{\sin(2x + 2y) - \cos y \sin y}.$$

7. Find y' if $x = 3 + \sqrt{x^2 + y^2}$.

Solution: Taking the derivative gives $1 = \dfrac{2x + 2yy'}{2\sqrt{x^2 + y^2}}$ so solving gives

$$y' = \frac{\sqrt{x^2 + y^2} - x}{y}.$$

8. Given $\frac{x - y^3}{y + x^2} = x + 2$, find y'.

Solution: Multiply through by $(y + x^2)$ first and take the derivative of both sides to get $1 - 3y^2y' = xy' + y + 2y' + 3x^2 + 4x$ (you can do this without multiplying through but its a bit uglier). Then solving for y' gives

$$y' = \frac{1 - y - 3x^2 - 4x}{3y^2 + x + 2}.$$

9. Find y' give that $\frac{y}{x^3} + \frac{x}{y^3} = x^2y^4$.

Solution: Multiply through by x^3y^3 to clear denominators so $y^4+x^4 = x^5y^7$. Taking the derivative gives $4y^3y' + 4x^3 = 7x^5y^6y' + 5x^4y^7$. Solving for y' gives

$$y' = \frac{5x^4y^7 - 4x^3}{4y^3 - 7x^5y^6}.$$

10. Find y' given $(x^2 + y^2)^3 = 8x^2y^2$.

Solution: Take the derivative and solve for y' to get

$$y' = \frac{16xy^2 - 6x(x^2 + y^2)^2}{6y(x^2 + y^2)^2 - 16x^2y}.$$